MATHEMATICAL MODELING TEACHER PREPARATION BASED ON MULTIPLE EXPERIENCES

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MAA MATHEMATICAL ASSOCIATION OF AMERICA

MATHFEST

Tampa, FL
August 2-5, 2023

THANK YOU TO SESSION ORGANIZERS

MATHEMATICAL MODELING WITH PRE-SERVICE AND IN-SERVICE TEACHERS

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PREPARING TEACHERS IN MATHEMATICAL MODELING

CCSSM (2010) – initiated modeling in K-12

K-12 ***Mathematical Practice* -** MP4. Model with mathematics

HS ***Conceptual Category*** - modeling standards are dispersed throughout all other conceptual categories: number & quantity, algebra, functions, geometry, statistics & probability).

AMTE's *Standards for Preparing Teachers of Mathematics* (2017) advocate for modeling in teacher preparation.

Teacher preparation programs are inconsistent with requiring a course in modeling.

THE UNIVERSITY OF ARIZONA
COLLEGE OF SCIENCE
Secondary Mathematics Education Program

B.S./B.A. Mathematics degree program

ME is an option in the major.

Core courses in the major:

- Calculus series, linear algebra, diff. eq., statistics, geometry, number theory, proof, history of math, synthesis course
- No MM course

In general, PTs typically have little to no experience with modeling.

Our approach is to infuse mm in content and pedagogy courses - to expose PTs with multiple experiences.

Our research: curriculum development and work with PTs for developing competency and MMKT.

RESEARCH

Key research questions:

- How do pre- and in-service teachers build competency in MM?
- How do pre- and in-service teachers develop MMKT?

- **Analyses** – teachers' internalization of the modeling process and their self-created models provide insights into their modeling competency development.

- **Through metacognitive reflections** – PTs describe how they translate the modeling process into practice by focusing on the modeling process as they engage in modeling (Anhalt & Cortez, 2016).

- **Through simulations of practice (SOP)** (Grossman et al., 2009), PTs respond to student ideas/approaches to MM problems that helps develop their MMKT (Anhalt, Cortez, Kohler, & Tidwell, 2022).

- This is a process that takes time and experience in MM.

OUR GENERAL APPROACH IN WORKING WITH PST

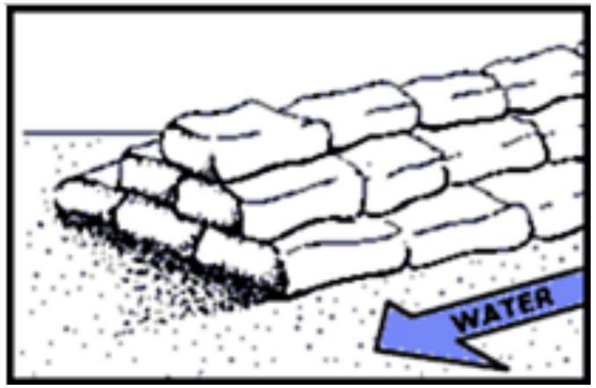
- Modeling is presented in a natural progression with a focus on elements of modeling
  - Simplifying the problem situation
  - Researching information
  - Prioritizing variables
  - Making assumptions
  - Create simple models
- Introductory tasks involve more guidance and structure for entering the modeling process
- Later experiences provide students opportunities for more independent exploration and decision making

PTs develop modeling competency through an experiential approach – they create models to solve problems while cognizant of the modeling process.

EXAMPLE TASK: FIGHTING FLOODS WITH SANDBAGS

The table shows estimates to build sandbag walls that are **100 feet long** and various heights.

- Why do you think their estimates are so different?
- Develop your own procedure to estimate the number of sandbags.
- How do your estimates compare with theirs?



Height of sandbag wall	Army Corps of Engineers estimate	Missouri Dept. of Natural Resources estimate	Your estimate
1 foot	600 bags	500 bags	
2 feet	2,100 bags	1,000 bags	
3 feet	4,500 bags	2,100 bags	
4 feet	7,800 bags	3,600 bags	
5 feet	-	5,500 bags	

BUILDING BACKGROUND KNOWLEDGE

Why Sandbags?

1. **Easy to use**
2. **Inexpensive**
3. **They work!**

science.howstuffworks.com

Sandbags stacked in a pyramid formation.

Keep Your Receipts

If you have flood insurance, the National Flood Insurance Program (NFIP) will give money back on many of your flood control expenses. Items like sandbags, lumber and water pumps may be reimbursable up to $1,000.

The New York Times

Published: October 13, 1874

LOUISIANA.

THE LEVEE SYSTEM OF THE STATE.

OVERFLOWS AND HOW THEY ARE PREVENTED—RIVER EMBANKMENTS BEFORE THE WAR AND NOW—THE LOUISIANA LEVEE COMPANY—ITS WORK AND PROFITS.

From Our Special Correspondent.

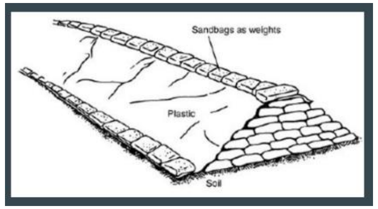
NEW-ORLEANS, La., Thursday, Oct. 8, 1874.

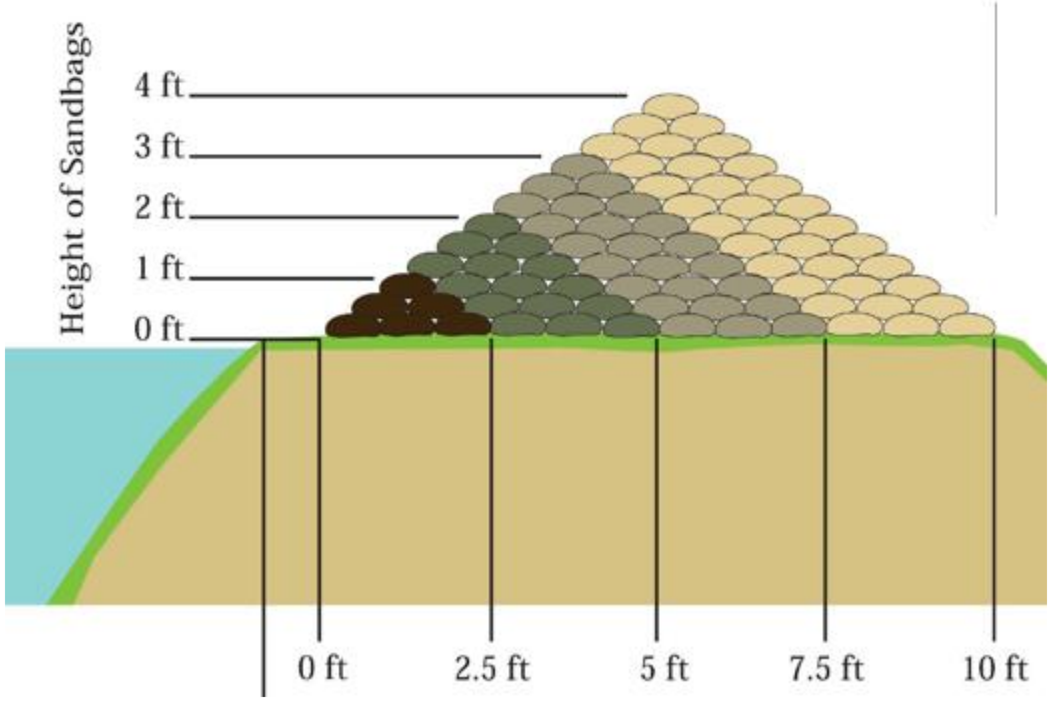
Recent events have directed the attention of the whole country to the political situation in Louisiana, and many earnest, thoughtful men are now endeavoring to suggest some means by

EXAMPLE APPROACHES

Typical assumptions:

- Filled sandbag length, height
- Stacked as in figure

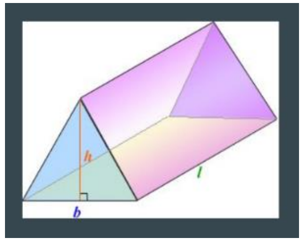




Volume approach:

- Estimate volume (V) of wall
- Estimate volume (v) of 1 sandbag

N = V/v

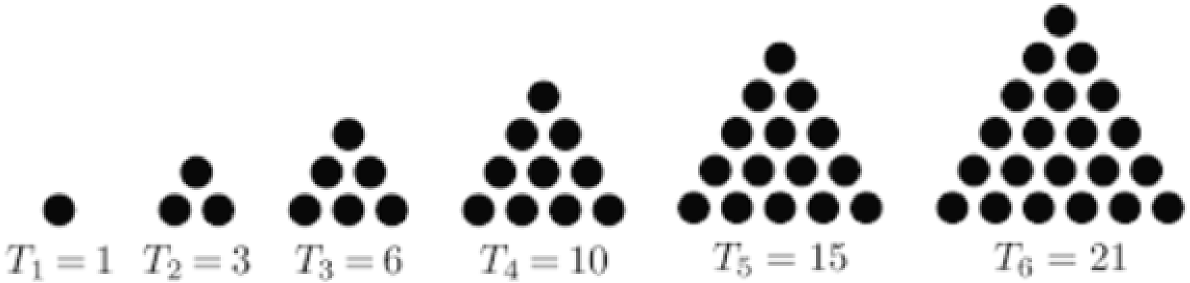


Counting approach:

- Estimate the number of layers, L
- N = 100 (1 + 2 + 3 + ... + L)

$$\sum_{k=1}^{L} k$$

Connection to triangular numbers



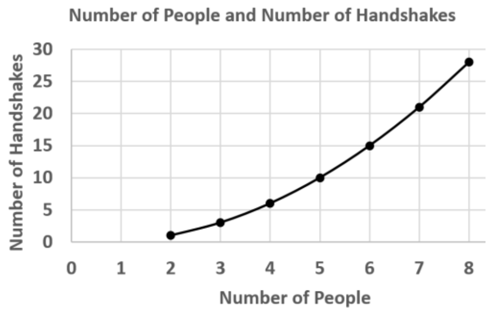
MATHEMATICAL CONNECTIONS TO OTHER PROBLEMS
The Handshake Problem

- Sometimes people shake hands to greet one another. Assuming all people in a room shake hands with everyone, how many handshakes would be exchanged among 20 people?



- Solve the problem. Generalize - create a function for any number of people.
- What are the different ways to represent the solution?

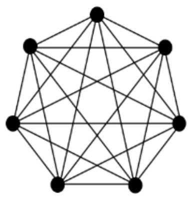
EXPLORATION WITH TRIANGULAR NUMBERS



Graphical representation

$$F(n) = \frac{n(n-1)}{2}$$

Function notation



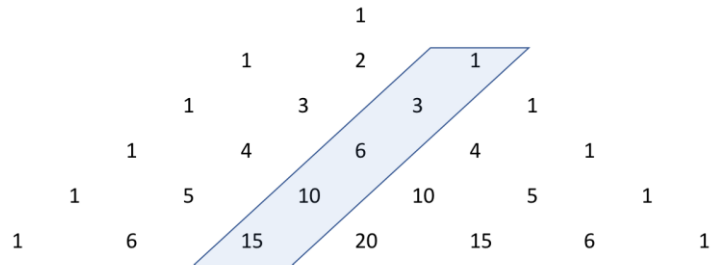
Representation from graph theory

6						
5	5					
4	4	4				
3	3	3	3			
2	2	2	2	2		
1	1	1	1	1	1	

Geometric representation
6+5+4+3+2+1

Number of People (n)	Number of Handshakes
1	0
2	1
3	3
4	6
5	10
n	?

Data points



Triangular numbers in Pascal's Triangle

$$\frac{n(n-1)}{2} = \frac{n!}{2!\,(n-2)!} = \binom{n}{2}$$

Combinatorial approach

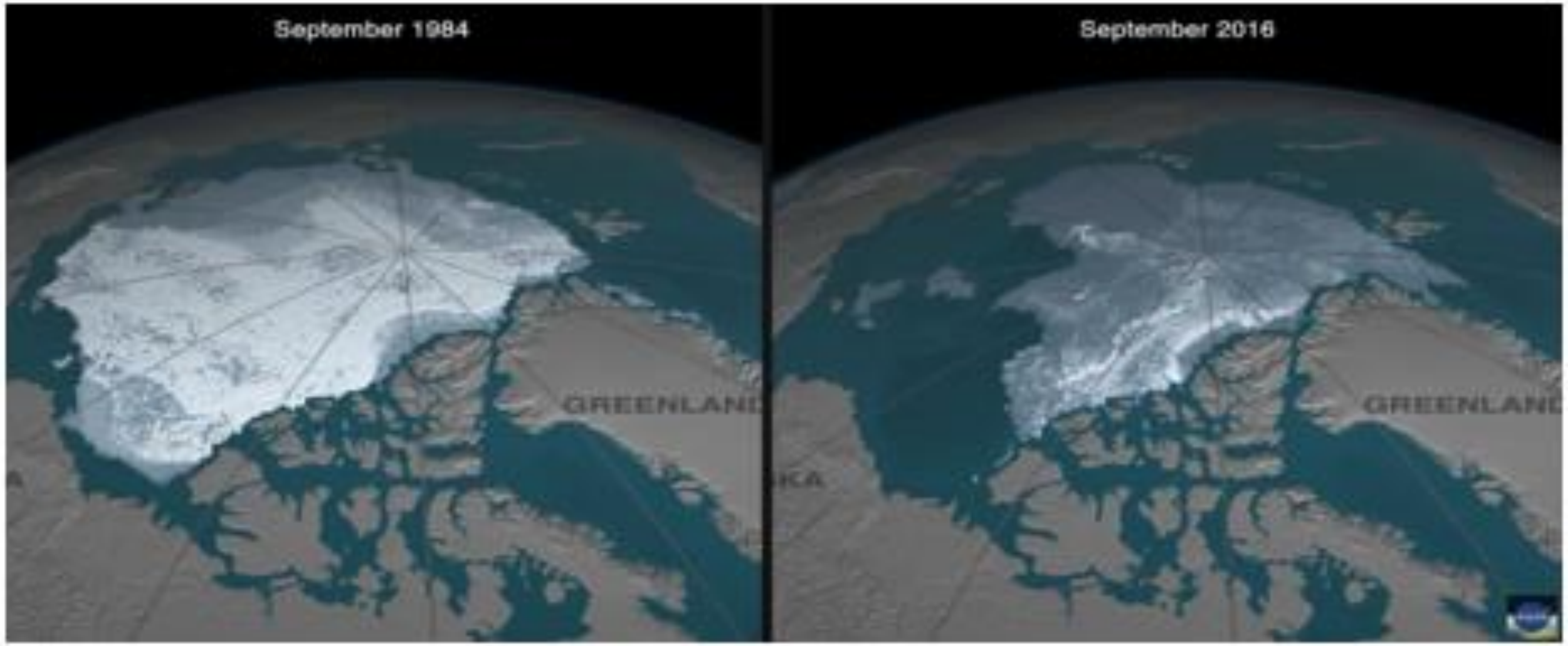
Connection to linear algebra

$$\begin{pmatrix} a1 & b1 & c1 & d1 & e1 & f1 \\ 0 & a2 & b2 & c2 & d2 & e2 \\ 0 & 0 & a3 & b3 & c3 & d3 \\ 0 & 0 & 0 & a4 & b4 & c4 \\ 0 & 0 & 0 & 0 & a5 & b5 \\ 0 & 0 & 0 & 0 & 0 & a6 \end{pmatrix}$$

Upper triangular matrix

Example Task: Arctic Sea Ice Region in 1984 and 2016

Below are NASA images of the Earth’s Arctic Polar Ice regions in 1984 and 2016.

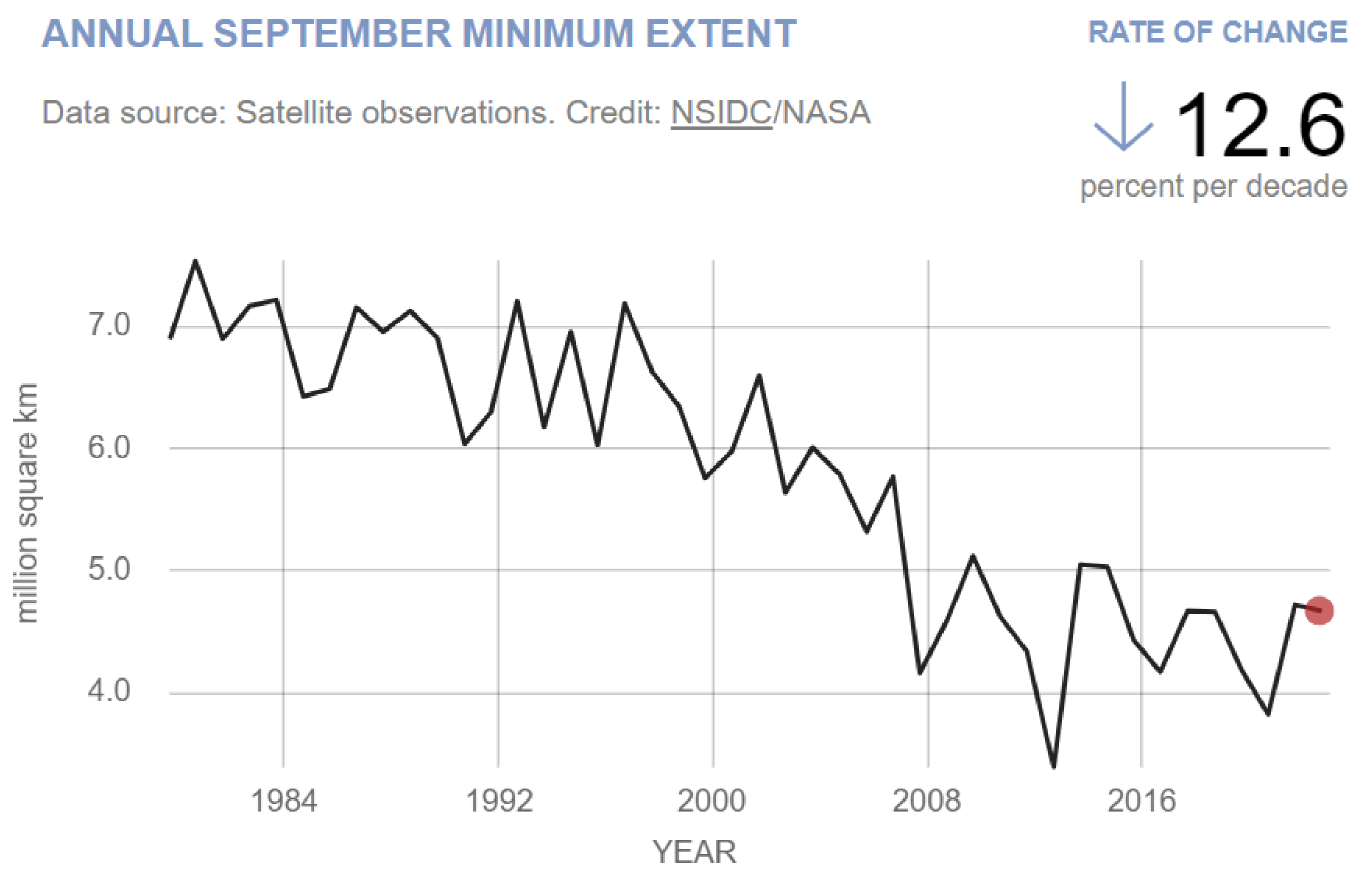


Develop a method that can be used to approximate the area of the Arctic Sea ice based on these satellite images.

Using your method, determine the percentage decrease in the Arctic Sea ice from 1984 to 2016.

Example Task: NASA Data on Arctic Sea Ice Extent Decling

https://climate.nasa.gov/vital-signs/arctic-sea-ice/



The NASA states that "September Arctic Sea ice is now declining at a rate of 12.6% per decade, relative to the 1981 to 2010 average." Do you agree or disagree with that statement? Perform and show calculations that support your conclusion.

1981 - 1990

Year	Ice extent in million sq km
1981	6.9
1982	7.17
1983	7.22
1984	6.43
1985	6.49
1986	7.16
1987	6.96
1988	7.13
1989	6.91
1990	6.04

1991-2000

Year	Ice extent in million sq km
1991	6.3
1992	7.21
1993	6.18
1994	6.96
1995	6.03
1996	7.19
1997	6.63
1998	6.35
1999	5.76
2000	5.98

2001-2010

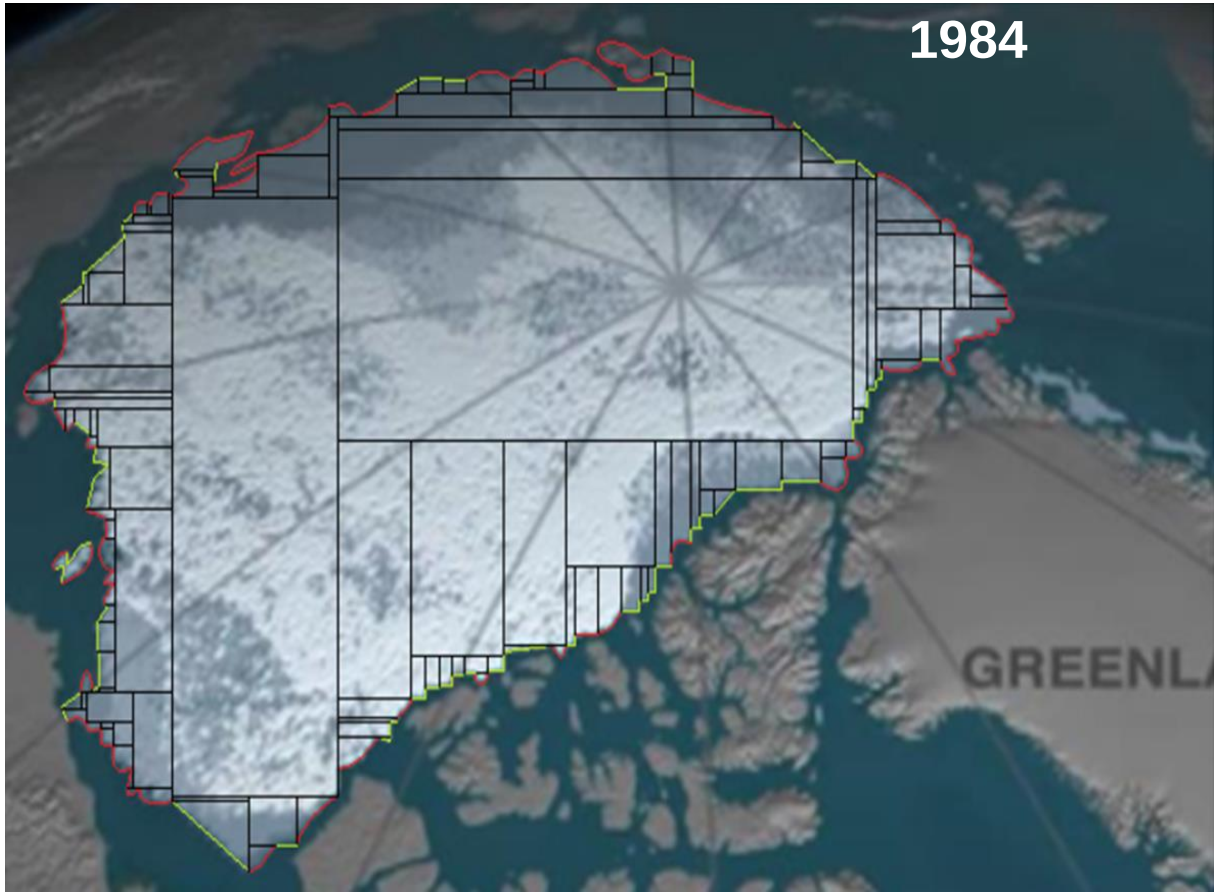
Year	Ice extent in million sq km
2001	6.6
2002	5.64
2003	6.01
2004	5.79
2005	5.32
2006	5.77
2007	4.16
2008	4.59
2009	5.12
2010	4.62

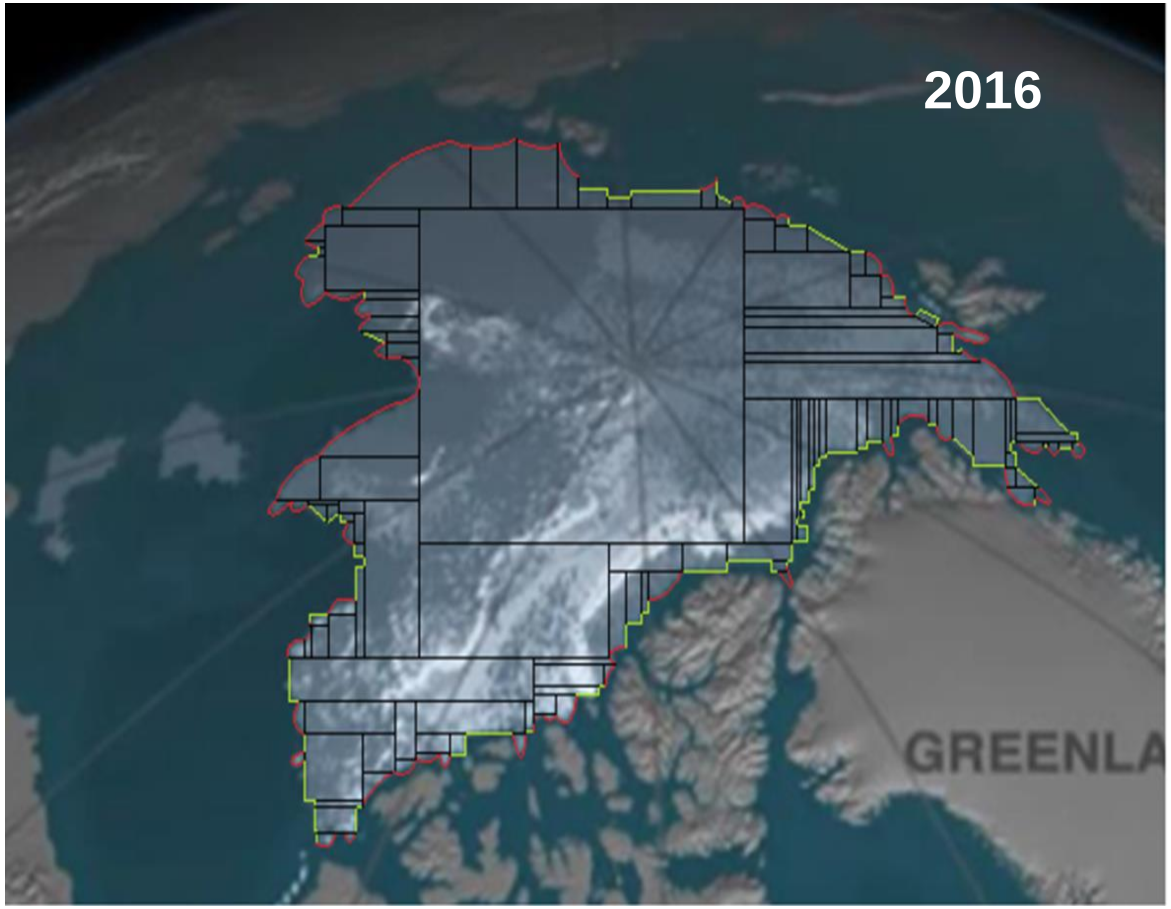
2011-2021

Year	Ice extent in million sq km
2011	4.34
2012	3.39
2013	5.05
2014	5.03
2015	4.43
2016	4.17
2017	4.67
2018	4.66
2019	4.19
2020	3.82
2021	4.72
2022	4.67

Example PT Approach

According to our solution, the percentage of ice in 2016 that was still existent compared to in 1984 is 821/1234 = 66.53%, which means that there is roughly 33.47% less ice in the 2016 image than in the 1984 image.





PROFESSIONAL DEVELOPMENT: WESTERN REGIONAL NOYCE CONFERENCE

240 Noyce Scholars and Fellows

14 states represented

- Undergraduate science & math PTs
- Graduate students, PTs
- In-service teachers in early career

- Introduction to Modeling
- Collaborative learning community
- Discussions, problem solving
- Posters: models created from NASA data on the Arctic Sea ice melting

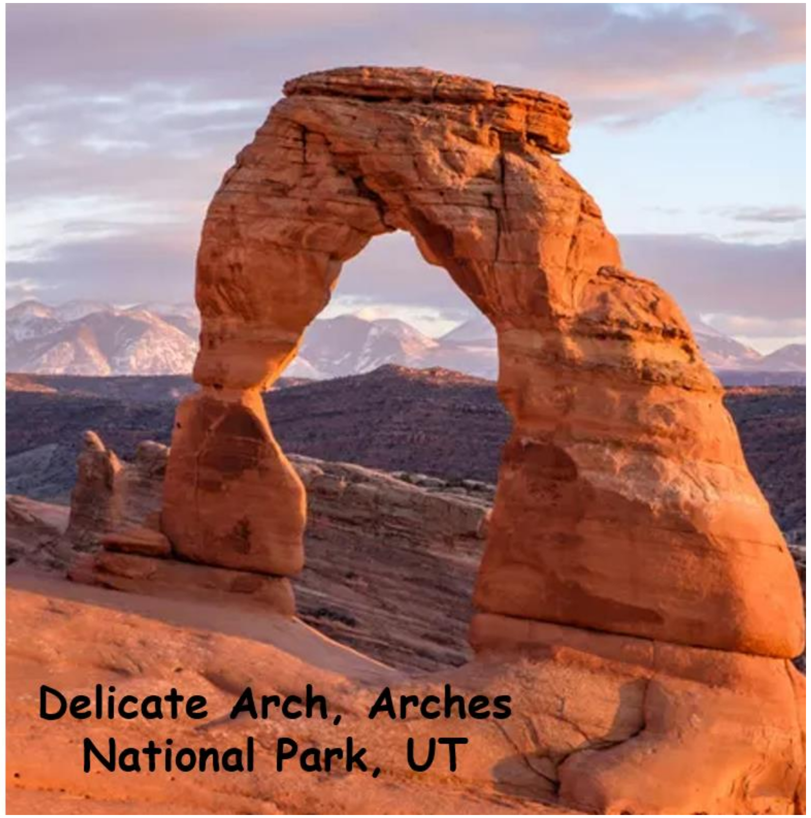
PROFESSIONAL DEVELOPMENT: LOCAL COMMUNITY MODELING

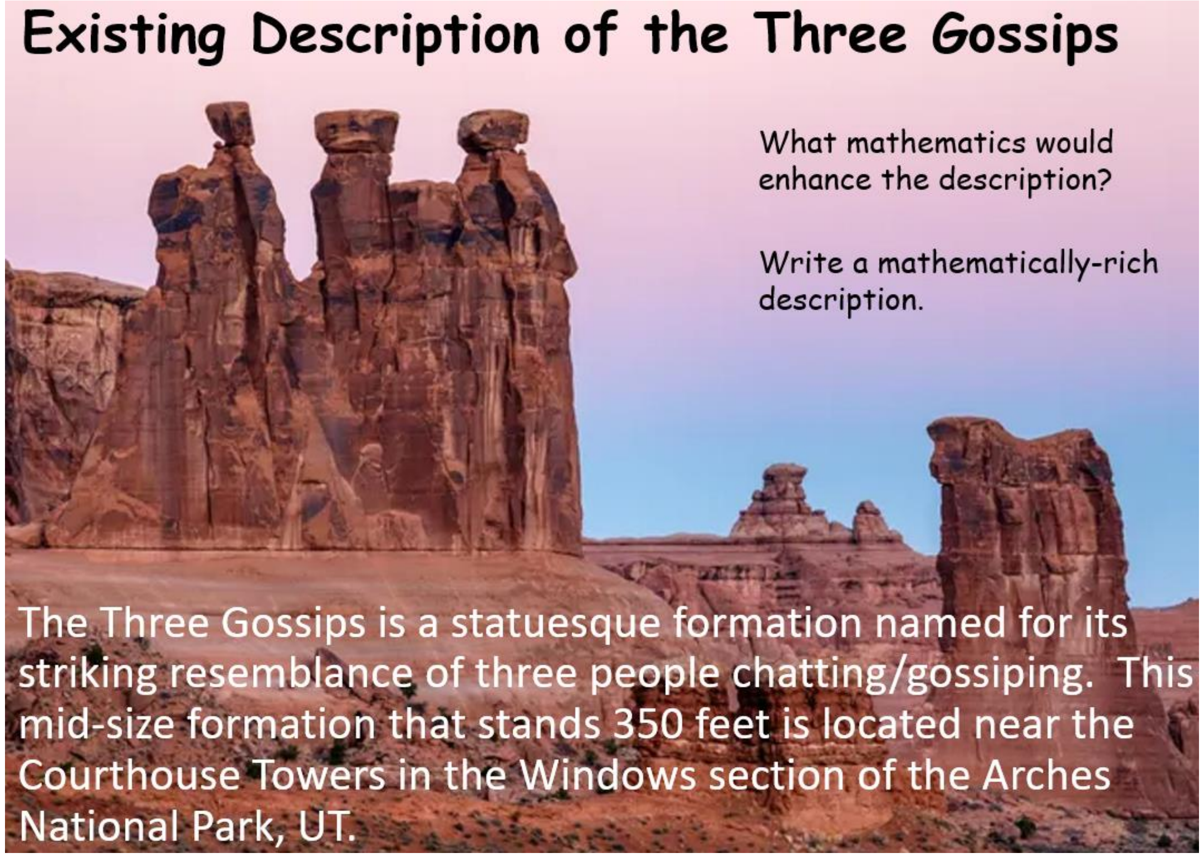
USU STEAM Expo+ Teacher Workshop, Blanding, UT

Discuss in small groups:
- What do you notice?
- How was this formed?
- How would you describe it geometrically?
- What is the name of it?
- Where is it located?

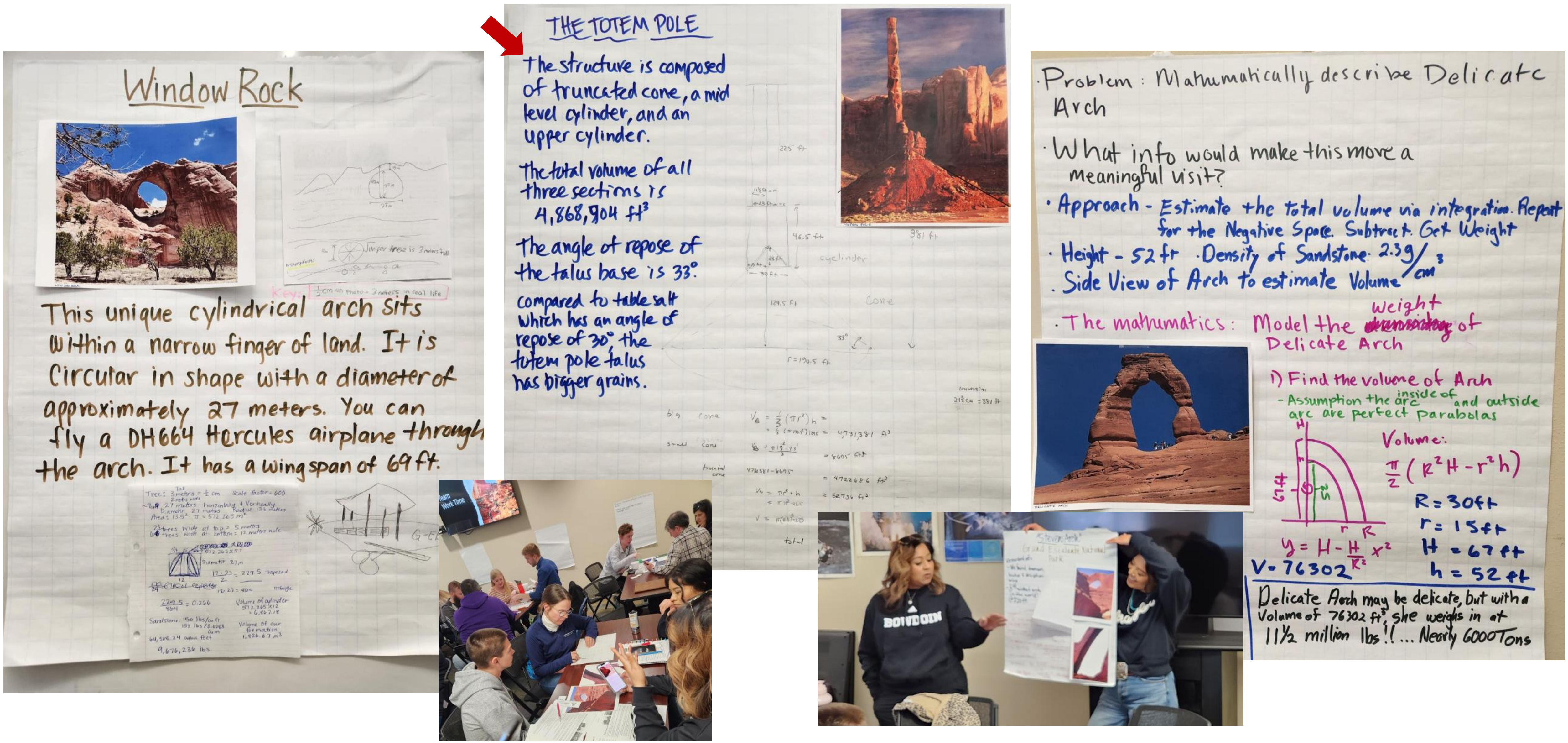
Group Work

Create a description of the rock formation through mathematical narrative. For example, include the geometric shapes, a maximum point, and unusual 3-dimensional widths, circumferences.



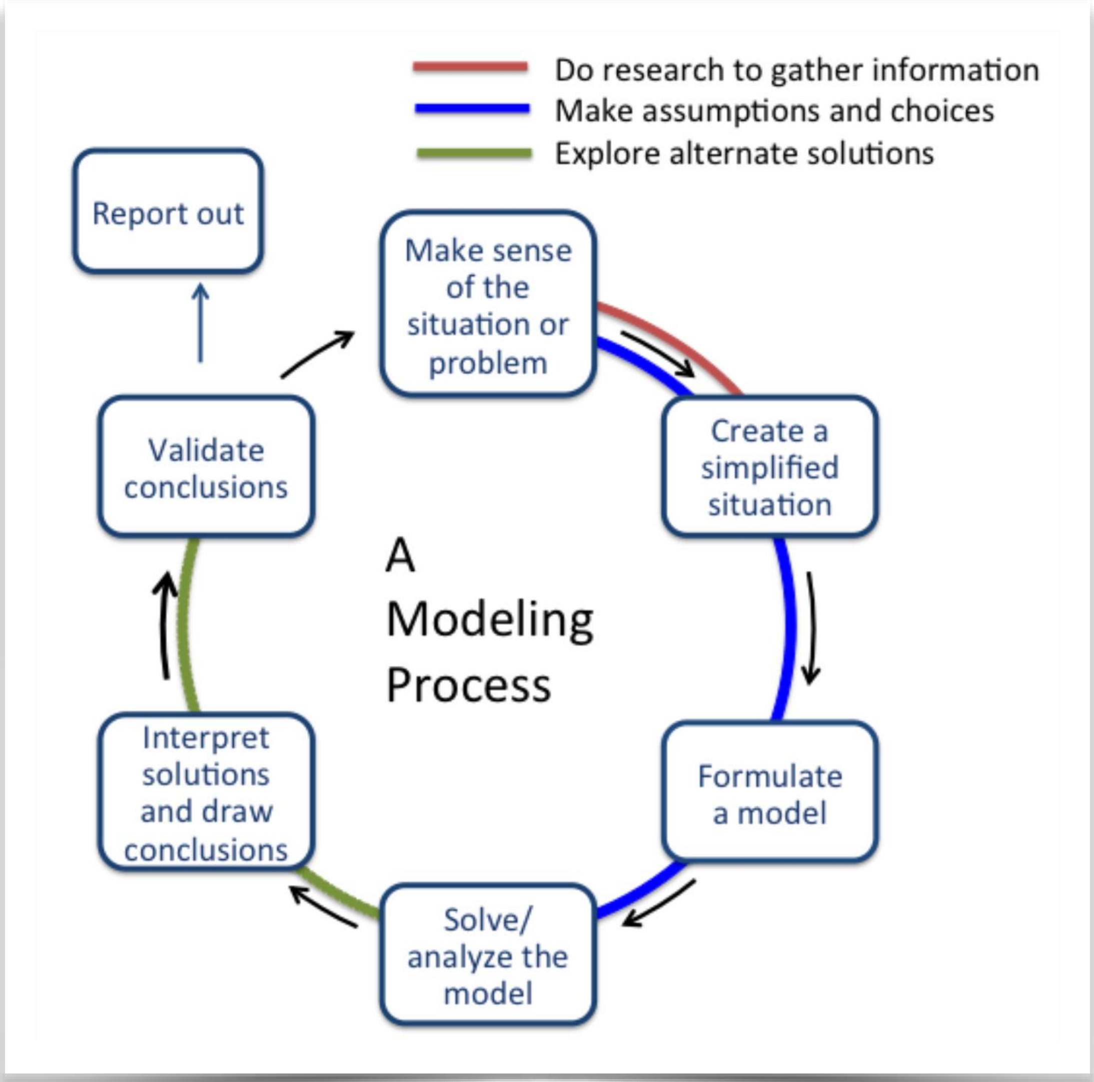


SAMPLE MODELS AND MATHEMATICAL DESCRIPTIONS OF ROCK FORMATIONS



REFLECTION ON THE MODELING PROCESS

- Consider the modeling process, its purpose, and the process of creating models.
- Which elements of modeling are clear to you, and which are less clear as you engage in modeling? Explain.
- Which aspects of modeling do you think secondary students would find relevant, and which would they find challenging? Explain.



MODULE(S²)

Mathematics Of Doing, Understanding, Learning and Educating for Secondary Schools

The Larger Project

Secondary Mathematics Teacher Preparation Curriculum

- Algebra
- Geometry
- **Mathematical Modeling**
- Statistics

Multi-institutional IUSE project

modules2.com

Award 1726707

OVERVIEW OF COURSE MATERIALS

modules2.com

MODULE(S²)
Mathematics Of Doing, Understanding, Learning and Educating for Secondary Schools

Module 1	Module 2	Module 3
The Process and Purpose of Mathematical Modeling	**Incorporating Real Data in Mathematical Modeling**	**Diverse Perspectives in Mathematical Modeling**
1. Ways of Thinking for Mathematical Modeling 2. Fighting Floods with Sandbags 3. Elements of the MM Process 4. Predicting the Evolution of STDs 5. Water Conservation 6. Analyzing Modeling Tasks: Rolling Cups 7. Critical Reading of Models: Muffin Sale	1. The Area of Tree Leaves 2. Cooling Coffee 3. Memorization 4. Pain Medication 5. Leaky Bucket 6. The Lost Cell Phone 7. The Trapeze and the Pendulum	1. Area of Sioux Reservation Land 2. Thermoclines and Air Pollution 3. Energy Saving Light Bulbs 4. Flint Water Crisis 5. SIR Disease Transmission Models Final Project

Non-traditional approach to teaching modeling; student-centered curriculum in which PTs create models

NSF Award 1726707

CLOSING THOUGHTS

- Developing MMK and MMKT requires deliberate integration of MM into teacher preparation coursework.
- PTs benefit from multiple exposures in MM.
- PTs should experience MM as learners, and then extend further to a professional level in the context of teaching and learning.

MATHEMATICAL MODELING TEACHER PREPARATION BASED ON MULTIPLE EXPERIENCES

MODULE(S^2)
Mathematics Of Doing, Understanding, Learning and Educating for Secondary Schools

modules2.com

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THE UNIVERSITY OF ARIZONA | COLLEGE OF SCIENCE MATHEMATICS